Models of Set Theory II - Winter 2013

Prof. Dr. Peter Koepke, Dr. Philipp Schlicht — Problem sheet 3

Problem 9 (4 Points)**.** A forcing $\mathbb{P} = (\mathbb{P}, \leq)$ is *κ-distributive* if the following condition holds: if $(U_\alpha)_{\alpha<\kappa}$ is a sequence of open dense subsets of $\mathbb{P}$, then $\bigcap_{\alpha<\kappa} U_\alpha$ is dense in $\mathbb{P}$. Suppose that M is a ground model, $(\mathbb{P}, \leq) \in M$ is a partial order, and κ is an infinite regular cardinal in M such that

$$M \models \text{"}(\mathbb{P}, \leq) \text{ is } \kappa\text{-distributive"}.$$

Suppose that G is $\mathbb{P}$-generic over M. Show that $f \in M$ for every function $f\colon \kappa \to M$ with $f \in M[G]$.

Problem 10 (4 Points)**.**

(a) Prove that $\mathbb{P} \times \mathbb{Q}$ is Knaster if $\mathbb{P}$ and $\mathbb{Q}$ are Knaster.

(b) Prove from MA_{ω_1} that $\mathbb{P} \times \mathbb{Q}$ is c.c.c if and only if $\mathbb{P}$ and $\mathbb{Q}$ are c.c.c.

Problem 11 (6 Points)**.** Suppose that κ is an uncountable regular cardinal. Suppose that $\mathbb{P}$ is κ-closed and $\dot{\mathbb{Q}}$ is a $\mathbb{P}$-name such that $1_{\mathbb{P}} \Vdash$ "$\dot{\mathbb{Q}}$ is $\check{\kappa}$-closed". Show that $\mathbb{P} * \dot{\mathbb{Q}}$ is κ-closed.

Problem 12 (6 Points)**.** Suppose that M is a ground model. Suppose that

$$(\mathbb{P}_n, \leq_n, 1_{\mathbb{P}_n}, \dot{\mathbb{Q}}_m, \dot{\leq}_m)_{m<\omega,\ n\leq\omega}$$

is a finite support iteration in M with $\mathbb{P}_0 = Col(\omega, \omega_1)$ and $1_{\mathbb{P}_n} \Vdash_{\mathbb{P}_n} \dot{\mathbb{Q}}_n = Col(\omega, \omega_1)$ for all $n < \omega$. Suppose that G is $\mathbb{P}_\omega$-generic over M. Decide which infinite cardinals κ of M are cardinals in $M[G]$.

Please hand in your solutions on Monday, November 11 before the lecture.